vvt
NEET | IIT - JEE
FOUNDATIONS

NEET2025

# PHYSICS NOTES

## Motion in a Plane

# MOTION IN A PLANE

**Oblique Projectile:**

Any body projected into the axis at an angle other than $90°$ with the horizontal near the surface of the Earth, is called a Projectile.

The science of projectile motion is called Ballistics.

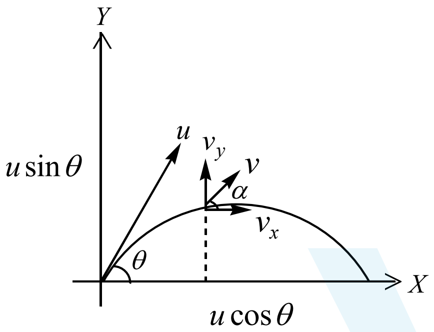
**Examples of Projectiles:**

i. A cricket ball thrown by a fielder.

ii. A bullet fired from a gun.

iii. A javelin thrown by an athlete.

iv. A jet of water from a rubber tube impelled into air.




* It is a body projected other than $90°$

* Initial velocity of a projection $\vec{u} = u_x\hat{i} + u_y\hat{j}$

$\vec{u} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

* Acceleration of a projective $\vec{a} = a_x\hat{i} + a_y\hat{j}$

$a_x = 0;\ a_y = -g;\ \vec{a} = a_x\hat{i} + a_y\hat{j}$

* As there is no effect on $u\cos\theta$, it remains constant throughout the flight.

* As $u\sin\theta$ direction opposite to '$g$.' So it's vertical speed decreases and becomes zero at the highest point.

* At any instant of time '$t$' velocity of a projective $\vec{v} = v_x\hat{i} + v_y\hat{j}$

$v_x = v_y = u\cos\theta$

$v_y = u_y + a_y t = (u\sin\theta) - gt$

$\therefore\ \vec{v} = (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}$

$|\vec{v}| = \sqrt{v_x^2 + v_y^2}$

$|\vec{v}| = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$

* It's direction making an angle with horizontal

$$\text{Tan}\,\alpha = \frac{v_y}{v_x} = \frac{(u\sin\theta - gt)}{(u\cos\theta)}$$

$$\alpha = \text{Tan}^{-1}\left(\frac{u\sin\theta - gt}{u\cos\theta}\right)$$

* It's displacement $\vec{s} = s_x\hat{i} + s_y\hat{j}$

Where $s_x = x$; $s_y = y$

$$x = (u\cos\theta)t \quad (\because a_x = 0)$$

$$s_y = (u\sin\theta)t - \frac{1}{2}gt^2 \quad (\because a_y = -g)$$

$$\vec{s} = (u\cos\theta)t\,\hat{i} + \left((u\sin\theta)t - \frac{1}{2}gt^2\right)\hat{j}$$

* In horizontal direction $x = (u\cos\theta)t$

$$t = \left(\frac{x}{u\cos\theta}\right)$$

In vertical direction $y = (u\sin\theta)t - \frac{1}{2}gt^2$

$$y = (u\sin\theta)\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$y = (\text{Tan}\,\theta)x - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$$

$$y = (\text{Tan}\,\theta)x - \left(\frac{g}{2u^2\cos^2\theta}\right)x^2$$

$$\boxed{y = Ax - Bx^2}$$

Where $A = \text{Tan}\,\theta$; $B = \left(\frac{g}{2u^2\cos^2\theta}\right)$

(i) Time of ascent = Time of descent

$$\boxed{t_a = t_d = \frac{u_y}{g} = \frac{u\sin\theta}{g}}$$

(ii) Time of flight $\boxed{T = 2t_a = 2t_d = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}}$

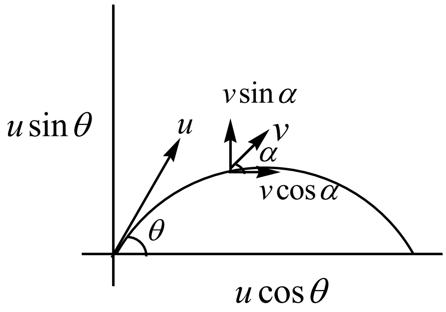
(iii) Maximum height $\boxed{H = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g} = \frac{A^2}{4B}}$

(iv) Range $\boxed{R=\frac{2u_x u_y}{g}=\frac{u^2\sin 2\theta}{g}=\frac{A}{B}}$

(v) Relation between $H$ & $R$ is $\frac{H}{R}=\frac{\tan\theta}{u}$

(vi) Relation between $H$ & $T$ is $H=\frac{1}{8}gT^2$

* A body is projected with speed '$u$' at an angle '$\theta$' with horizontal. If it makes an angle '$\alpha$' with the horizontal, then it's speed at that instant will be




In horizontal direction $u\cos\theta=v\cos\alpha$

$$v=\frac{u\cos\theta}{\cos\alpha}=u\cos\theta\sec\alpha$$

$$v=u\cos\theta\cdot\sec\alpha$$

* A particle is projected at angle '$\alpha$' with the horizontal. After time '$t$' it appears to have an angle of deviation '$\beta$' as seen from point of projection, then initial velocity

$$\text{Tan}\,\beta=\frac{y}{x}$$

$$\text{Tan}\,\beta=\frac{(u\sin\alpha)t-\frac{1}{2}gt^2}{(u\cos\alpha)t}$$

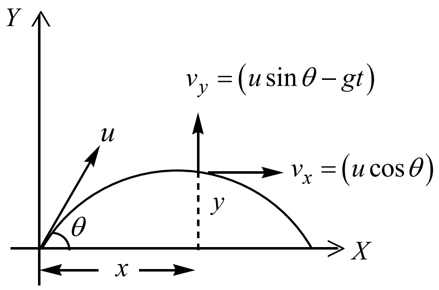
$$u\cos\alpha\tan\beta=u\sin\alpha-\frac{gt}{2}$$

$$\frac{gt}{2}=u\left[\sin\alpha-\cos\alpha\cdot\frac{\sin\beta}{\cos\beta}\right]$$

$$\frac{gt}{2}=u\left[\frac{\sin(\alpha-\beta)}{\cos\beta}\right]$$

$$\boxed{u=\frac{gt\cos\beta}{2\sin(\alpha-\beta)}}$$

* A particle of mass '$m$' is projected with speed '$u$' at angle '$\theta$' with the horizontal. It's angular momentum about point of projection at any instant.




$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\vec{r} = (u\cos\theta)t\,\hat{i} + \left(u\sin\theta\, t - \frac{1}{2}gt^2\right)$$

$$\vec{v} = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\hat{j}$$

$$\vec{L} = m(\vec{r}\times\vec{v})$$

$$\vec{L} = m\begin{vmatrix} i & j & k \\ u\cos\theta\, t & u\sin\theta\, t - \frac{1}{2}gt^2 & 0 \\ u\cos\theta & u\sin\theta - gt & 0 \end{vmatrix}$$

$$\vec{L} = m[(0-0)\hat{i} - \hat{j}(0-0) + \hat{k}(u\cos\theta\, t(\sin\theta - gt)) - (u\cos\theta)\left(u\sin\theta\, t - \frac{1}{2}gt^2\right)$$

$$\vec{L} = m\left[u^2\sin\theta\cdot\cos\theta\, t - u\cos\theta\, t^2 - u^2\sin\theta\cos\theta\, t + u\cos\theta\frac{gt^2}{2}\right]$$

$$\vec{L} = m\left(-u\cos\theta\frac{gt^2}{2}\right)k$$

$$\vec{L} = \frac{1}{2}(mu\cos\theta)gt^2(-\hat{k})$$

$$L \propto t^2$$

$\therefore$ Angular momentum continuously increases till it reaches the ground.

**Torque acting on a projectile:**

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\vec{r} = (u\cos\theta\, t)\hat{i} + \left[u\sin\theta\, t - \frac{1}{2}gt^2\right]\hat{j}$$

$$\vec{\tau} = \vec{r}\times\vec{F} = (mg)u\cos\theta\, t\left(-\hat{k}\right)$$

$$\therefore\ \vec{\tau} = (mu\cos\theta)gt\left(-\hat{k}\right)$$

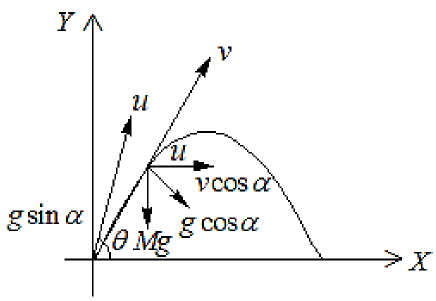
$$\vec{\tau} = (mu\cos\theta)gt$$

$\boxed{\tau \propto t}$

$\therefore$ Torque continuously increases till it reaches the ground.

**Radius of curvature of a projectile:**

Let '$v$' be the velocity of projectile when it makes '$\alpha$' with horizontal then




$v\cos\alpha = u\cos\theta$

$$\boxed{v = \frac{u\cos\theta}{\cos\alpha}}$$

Where '$g\cos\alpha$' plays the role of the radial acceleration.

$$g\cos\alpha = \frac{v^2}{R}$$

$$R = \frac{v^2}{g\cos\alpha}$$

$$R = \left[\frac{u^2\cos^2\theta}{\cos^2\alpha}\right]\left[\frac{1}{g\cos\alpha}\right]$$

$$\boxed{\therefore\ R = \frac{u^2\cos^2\theta}{g\cos^2\alpha}}$$

When the projectile is at height point $(\alpha = 0)$

$$\boxed{\therefore\ R = \frac{u^2\cos^2\theta}{g}}$$

$\rightarrow$ A particle projected with speed '$u$' at an angle '$\theta$' with the horizontal after time '$t$' which is perpendicular to initial velocity vector

$\vec{u} = (u\cos\theta)\hat{i} + (u\sin\theta)\hat{j}$

After '$t$' sec

$\vec{v} = (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}$

As $\vec{u} \perp \vec{v}$ so $\bar{u}\cdot\bar{v} = 0$

$u^2\cos^2\theta + u^2\sin^2\theta - u\sin\theta\, gt = 0$

$u^2 = (u\sin\theta)\, gt$

$$\boxed{t = \frac{u}{g\sin\theta}}$$

Limits of '$\theta$', here $t \le T$

$$\frac{n}{g\sin\theta} \le \frac{2u\sin\theta}{g}$$

$$\frac{1}{\sin^2\theta} \le 2$$

$$\sin^2\theta > \frac{1}{2}$$

$$\sin\theta \ge \frac{1}{\sqrt{2}}$$

$$\boxed{\theta \ge 45^\circ}$$

$\therefore$ Limits $= 45^\circ \le \theta \le 90^\circ$

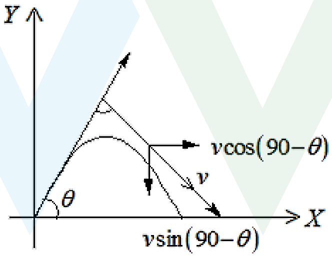
**Note:**

In the above case magnitude of its velocity perpendicular to initial velocity vector, then horizontal component is constant.

$$u\cos\theta = v\cos(90-\theta)$$

$$u\cos\theta = v\sin\theta$$

$$\boxed{v = u\cot\theta}$$




**Characteristics of a projectile:**

$\rightarrow$ Time of flight $T = \dfrac{2u\sin\theta}{g} = \dfrac{2u_y}{g}$ (or) $A\sqrt{\dfrac{2}{\Delta g}}$

$\rightarrow$ Range of a projectile $R = \dfrac{u^2\sin 2\theta}{g} = \dfrac{2u_x u_y}{g}$ (or) $\dfrac{A}{B}$

$\rightarrow$ Max. height $H = \dfrac{u^2\sin^2\theta}{2g} = \dfrac{A^2}{4B}$ (or) $\dfrac{u_y^2}{2g}$

$\rightarrow$ Relation between range and max. height

$$\frac{R}{H} = \frac{4}{\text{Tan}\,\theta}$$

$$\boxed{R\,\text{Tan}\,\theta = 4H}$$

$\rightarrow$ Relation between max. height and time of flight

$$\frac{H}{T^2} = \frac{\dfrac{u^2\sin^2\theta}{2g}}{\dfrac{4u^2\sin^2\theta}{g^2}}$$

$$\frac{H}{T^2}=\frac{g}{8}$$

$$\boxed{H=\frac{gT^2}{8}}$$

$\rightarrow$ A particle is projected at angle '$\theta$' with the horizontal. If the angle of elevation of highest point as seen from the point of projection is '$\phi$' then $\text{Tan}\,\phi=?$

$$\text{Tan}\,\phi=\frac{H}{(R/2)}=\frac{2H}{R}$$

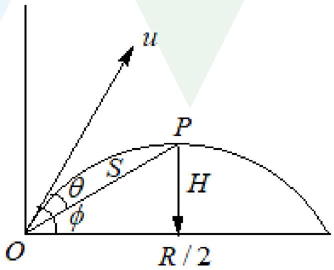
But $R\,\text{Tan}\,\theta=4H$

$$\frac{\text{Tan}\,\theta}{4}=\left(\frac{H}{2}\right)$$

$$\text{Tan}\,\phi=2\left[\frac{\text{Tan}\,\theta}{4}\right]$$

$$\boxed{\therefore\ \text{Tan}\,\phi=\frac{\text{Tan}\,\theta}{2}}$$

$\rightarrow$ A particle is projected with speed '$u$' at an angle '$\theta$' with the horizontal, then the magnitude of average velocity during the time that reaches to the highest point.




$$S=\sqrt{(R/2)^2+H^2}$$

$$S=\sqrt{\frac{R^2}{4}+\left[\frac{R\,\text{Tan}\,\theta}{4}\right]^2}$$

$\therefore\ R\,\text{Tan}\,\theta=4H$

$$S=\sqrt{\left[\frac{R^2}{4}+\frac{R^2\,\text{Tan}\,\theta}{16}\right]}$$

$$V_{\text{avg}}=\frac{S}{(T/2)}=\frac{2}{T}\left[\sqrt{\frac{R^2}{4}+\frac{R^2\,\text{Tan}^2\,\theta}{16}}\right]$$

$$V_{\text{avg}}=\frac{2R}{2T}\sqrt{\left[1+\frac{\text{Tan}^2\,\theta}{4}\right]}$$

$$=\frac{(u\cos\theta)T}{T}\left[\sqrt{\frac{4+\text{Tan}^2\,\theta}{4}}\right]$$

$$=\frac{u\cos\theta}{2}\left(\sqrt{4+\text{Tan}^2\,\theta}\right)$$

$$V_{\text{avg}}=\frac{u\cos\theta}{2}\left[\sqrt{\frac{4\cos^2\theta+\sin^2\theta}{\cos^2\theta}}\right]=\frac{u}{2}\sqrt{3\cos^2\theta+\cos^2\theta+\sin^2\theta}$$

$$\boxed{\therefore\ V_{\text{avg}} = \frac{u}{2}\sqrt{1+3\cos^2\theta}}$$

* In the above case magnitude of avg. velocity for its whole journey

For total journey $S = R;\ t = T$

$$\therefore\ V_{\text{avg}} = \frac{R}{T} = \frac{(u\cos\theta)T}{T}$$

$$\boxed{\therefore\ V_{\text{avg}} = u\cos\theta}$$

* The horizontal & vertical displacement of a projectile are given by $x = at,\ y = bt - ct^2$. Then

- Velocity of projection $= \sqrt{a^2+b^2}$
- Angle of projection $= \text{Tan}^{-1}\left(\frac{a}{b}\right)$
- Acceleration of projection $= 2c$
- Maximum height $= \frac{b^2}{4c}$
- Horizontal range $= \frac{ab}{c}$

→ A body is projected with velocity of $U = U_x\,\hat{i} + U_y\,\hat{j} + U_z\,\hat{k}$ where $x,\ y$ are in horizontal plane, then

- Time of flight $T = \frac{2U_z}{g}$
- Max. height $H = \frac{U_z^2}{2g}$
- Range $R = \left(\sqrt{U_x^2+U_y^2}\right)\cdot\frac{2U_z}{g}$

→ For two angles with same range (complementary angles):

$$R_1 = R_2 = R = \frac{U^2\sin 2\theta}{g}$$

* $H_1 = \frac{U^2\sin^2\theta}{2g}$

$$H_2 = \frac{U^2\sin^2(90-\theta)}{2g}$$

$$H_2 = \frac{U^2\cos^2\theta}{2g}$$

$$H_1 + H_2 = \frac{U^2}{2g}$$

* $U\sin\theta=\sqrt{2gH_1}$; $U\cos\theta=\sqrt{2gH_2}$

$$R=U^2\cdot\frac{2\sin\theta\cos\theta}{g}=\frac{2\cdot(U\sin\theta)(U\cos\theta)}{g}$$

$$\boxed{R=4\sqrt{H_1H_2}}$$

$$T_1=\frac{2U\sin\theta}{g}$$

$$T_2=\frac{2U\sin(90-\theta)}{g}$$

* $U\sin\theta=\frac{gT_1}{2}$; $U\cos\theta=\frac{gT_2}{2}$

$$R=\frac{2(U\sin\theta)(U\cos\theta)}{g}=\frac{2\left(\frac{gT_1}{2}\right)\left(\frac{gT_2}{2}\right)}{g}$$

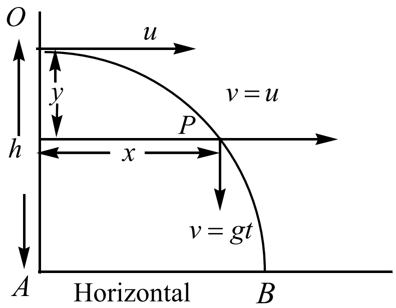
$$\boxed{R=\frac{1}{2}gT_1T_2}$$

## Horizontal Projectile:

### Equation for path (Trajectory):

Suppose a body is projected horizontally with an initial velocity '$u$' from the top of a tower of height '$h$' at time $t=0.$ As there is no horizontal acceleration, the horizontal velocity remains constant throughout the motion.

Hence after time '$t$' the velocity in horizontal direction will be $v_x=u$



Horizontal Projectile

Let the body reach a point '$p$' in time '$t$.; Let $x$ and $y$ be the co-ordinates of the body.

For the $y$ coordinate after time $t$ seconds

$$y=\frac{1}{2}gt^2 \quad \left[\because\ y=u_y t+\frac{1}{2}u_y t^2\right]\rightarrow(1)$$

For $x$ co-ordinate after $t$ seconds

$x=ut$

$\Rightarrow t=x/u\rightarrow(2)$

From eqn. (1) and (2) we get

$$y = \frac{1}{2} g \left[\frac{x}{u}\right]^2$$

$$\therefore\ y = \left[\frac{g}{2u^2}\right] x^2 \rightarrow (3)$$

$g$ and $u$ being constants $\left[\frac{g}{2u^2}\right]$ is a constant

If $\frac{g}{2u^2} = k$ then $y = kx^2$

This equation represents the equation of a parabola.

**Motion parameters of a horizontal projectile:**

**Time of Descent:**

It is the time the body takes to touch the ground after it is projectile from the height ‘ $h$ ’

For $y = h$ and $t = t_d$ we get

$$h = \frac{1}{2} g t_d^2$$

$$\therefore t_d = \sqrt{\frac{2h}{g}}$$

The time of descent is independent of initial velocity with which the body is projected and depends only on the height from which it is projected.

**Range:**

The maximum horizontal distance travelled by the body while it touches the ground is called range $(R)$.

It is shown as AB in the fig. As the horizontal velocity is constant

Range = Horizontal velocity × Time of descent

$$\Rightarrow R = (u) t_d$$

But $t_d = \sqrt{\frac{2h}{g}}$ and hence $R = (u)\sqrt{\frac{2h}{g}}$

**Velocity of the projectile at any time ‘$t$’:**

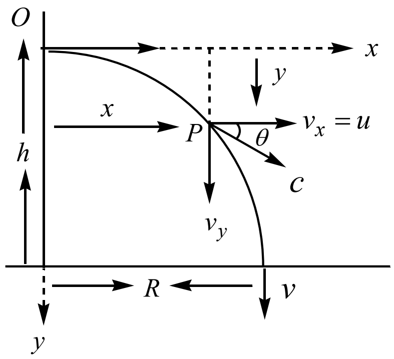
Let the body be at point ‘ $P$ ’ after time ‘ $t$ ’

Let $v_x$ *and* $v_y$ be velocities along $x$ and $y$ directions

The horizontal velocity remains constant throughout the motion. Hence $v_x = u$

The velocity along y-axis is $v_y = u_y + gt$ and $u_y = 0$ as the body is thrown horizontally initially.

$$\therefore v_y = gt$$



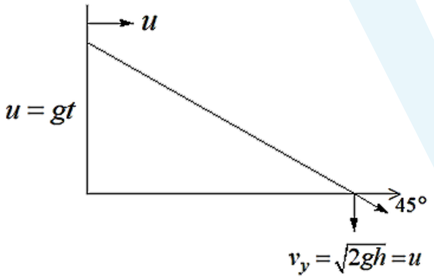

So the magnitude of the velocity $v=\sqrt{v_x^2+v_y^2}=\sqrt{u^2+g^2t^2}$

If velocity vector $\vec{v}$ makes an angle $\alpha$ with the horizontal then $\tan\alpha=\dfrac{v_y}{v_x}=\dfrac{gt}{u}$ $(\text{or})$ $\alpha=\tan^{-1}\left(\dfrac{gt}{u}\right)$

**Case-I:**

If a body projected horizontally with velocity $u$ from the top of a tower strikes the ground at an angle of $45^\circ \Rightarrow v_y=v_x$

$\therefore t=\dfrac{u}{g}$

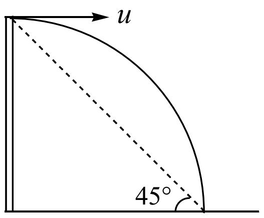



**Case-II:**

A body is projected horizontally from the top of a tower. The line joining the point of projection and the striking point make an angle of 450 with the ground. Then $h=u.$

$\dfrac{1}{2}gt^2=ut$

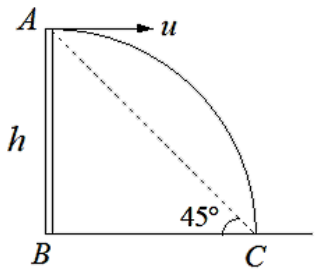
$t=\dfrac{2u}{g}$




**Case-III:**

A body is projected horizontally from the top of a tower. The line joining the point of projection and the striking point makes an angle of 450 with the ground. Then the displacement $=\sqrt{2}h$ $(\text{or})$ $\sqrt{2}x.$

From the figure, $\tan 45° = 1 = \dfrac{h}{x}$




$h = x$

$\therefore$ Displacement $AC = \sqrt{(AB)^2 + (BC)^2}$

$= \sqrt{h^2 + H^2} = \sqrt{2}h$ (or) $\sqrt{2}x$

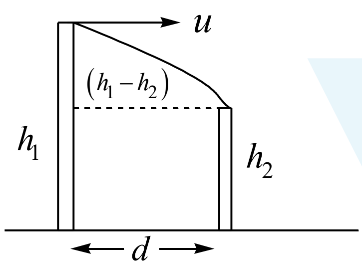
Since $h = x$

**Case-IV:**

Two towers having heights $h_1$ and $h_2$ are separated by a distance '$d$'

A person throws a ball horizontally with a velocity '$u$' from the top of the 1st tower to the top of the 2nd tower then

Time taken $t = \sqrt{\dfrac{2(h_1 + h_2)}{g}}$




Distance between the towers $d = ut = u\sqrt{\dfrac{2(h_1 - h_2)}{g}}$

**Case-V:**

An aeroplane flies horizontally with a velocity '$u$'

If a bomb is dropped by the pilot when the plane is at a height '$h$' then

a) The path of such as body is vertical straight line as seen by the pilot

b) The path is a parabola as seen by an observer on the ground

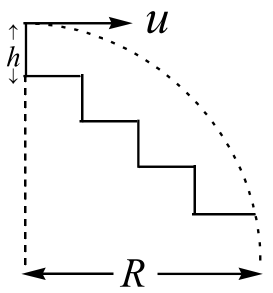
c) The body will strike the ground at a certain horizontal distance. This distance is equal to the range given by $x = ut = u\sqrt{\dfrac{2u}{g}}$

**Case-VI:**

A ball rolls off the top of a stair case with a horizontal velocity '$u$'. If each step has height '$h$' and width '$b$' then the ball will just hit the $n^{\text{th}}$ step if $n$ equals

$\therefore$ $nb = ut$ and $nh = \dfrac{1}{2}gt^2$

$n = \dfrac{2hu^2}{gb^2}$




**Case-VII:**

From the top of a tower one stone is thrown towards east with velocity $u_1$ and another is thrown towards north with velocity $u_2$ the distance between them after striking the ground

$$\boxed{d = t\sqrt{u_1^2 + u_2^2}}$$

**Case-VIII:**

Two bodies are thrown horizontally with velocities $u_1, u_2$ in mutually opposite directions from the same height. Then

a) Time after which velocity vectors are perpendicular is $t = \sqrt{\frac{u_1 u_2}{g}}$

For velocity vectors to be perpendicular after a time '$t$' their dot product must be zero

$$\therefore\ \overline{v_1} \cdot \overline{v_2} = 0$$

$$\left(u_1\,\hat{i} - gt\,\hat{j}\right) \cdot \left(-u_2\,\hat{i} - gt\,\hat{j}\right) = 0$$

$$\therefore\ t = \frac{\sqrt{u_1 u_2}}{g}$$

b) Separation between them when velocity vectors are perpendicular is

$$x = \left(u_1 + u_2\right)t = \frac{\left(u_1 + u_2\right)\sqrt{u_1 u_2}}{g}$$

c) Time after which their displacement vectors are perpendicular is $t = \frac{2\sqrt{u_1 u_2}}{g}$

For displacement vectors to be perpendicular then their dot product must be zero.

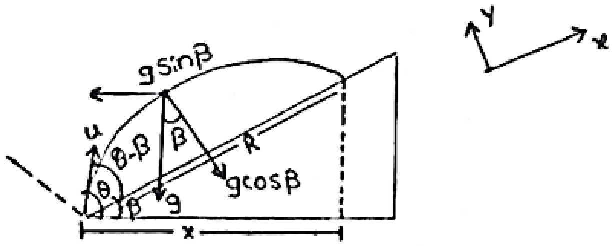
$$\left[u_1\,t\,\hat{i} - \frac{1}{2}gt^2\,\hat{j}\right] \cdot \left[-u_2\,t\,\hat{i} - \frac{1}{2}gt^2\,\hat{j}\right] = 0$$

$$\therefore\ t = \frac{2\sqrt{u_1 u_2}}{g}$$

d) Separation between them when displacement is perpendicular is

$$x = \left(u_1 + u_2\right)t = \frac{\left(u_1 + u_2\right)2\sqrt{u_1 u_2}}{g}$$

## Projectile motion on an inclined plane:




$\rightarrow u_x = u\cos(\theta-\beta);\ u_y = u\sin(\theta-\beta)$

$a_x = -g\sin\beta;\ a_y = -g\cos\beta$

$y = u\sin(\theta-\beta)t - \frac{1}{2}(g\cos\beta)t^2$

$\rightarrow$ If $t = T$ then $y = 0$, where $T$ = Time of flight

$0 = u\sin(\theta-\beta)T - \frac{1}{2}(g\cos\beta)T^2$

$u\sin(\theta-\beta)T = \frac{1}{2}(g\cos\beta)T^2$

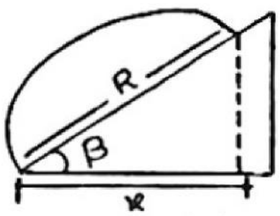
$$\boxed{T = \frac{2u\sin(\theta-\beta)}{g\cos\beta}}$$

$\rightarrow$ Range along the horizontal plane $x = (u\cos\theta)T$

$$\boxed{x = (u\cos\theta)\frac{2u\sin(\theta-\beta)}{g\cos\beta}}$$

$\rightarrow$ Range along inclined plane

$\cos\beta = \frac{x}{R} \Rightarrow \boxed{R = \frac{x}{\cos\beta}}$




$$\boxed{R = \frac{2u^2\cos\theta\sin(\theta-\beta)}{g\cos^2\beta}}$$

$[\because 2\cos A\sin B = \sin(A+B) - \sin(A-B)]$

$$R = \frac{u^2[\sin(\theta+\theta-\beta) - \sin(\theta-\theta+\beta)]}{g(1-\sin^2\beta)}$$

$$R = \frac{u^2(\sin(2\theta-\beta) - \sin\beta)}{g(1-\sin^2\beta)}$$

If $R_{\max}$ then $\sin(2\theta-\beta)$ is max

Then $\sin(2\theta-\beta)=1$

$$\boxed{\therefore R_{\max}=\frac{u^2(1-\sin\beta)}{g\left(1-\sin^2\beta\right)}}$$

$$R_{\max}=\frac{u^2(1-\sin\beta)}{g((1+\sin\beta)(1-\sin\beta))}$$

$$\boxed{R_{\max}=\frac{u^2}{g(1+\sin\beta)}}$$

(along the inclined plane in upwards)

**Vertical height at which projectile strikes:**

$$y=(u\sin\theta)T-\frac{1}{2}gT^2$$

$$y=(u\sin\theta)\left[\frac{2u\sin(\theta-\beta)}{g\cos\beta}\right]-\frac{1}{2}g\left[\frac{2u\sin(\theta-\beta)}{g\cos\beta}\right]^2$$

$$y=\frac{2u^2\sin\theta\sin(\theta-\beta)}{g\cos\beta}-\frac{2u^2\sin^2(\theta-\beta)}{g\cos^2\beta}$$

$$y=\frac{2u^2\sin(\theta-\beta)}{g\cos\beta}\left[\sin\theta-\frac{\sin(\theta-\beta)}{\cos\beta}\right]$$

$$y=\frac{2u^2\sin(\theta-\beta)}{g\cos\beta}\left[\frac{\sin\theta\cos\beta-\sin(\theta-\beta)}{\cos\beta}\right]$$

$$y=\frac{2u^2\sin(\theta-\beta)(\sin\theta\cos\beta-\sin\theta\cos\beta+\cos\theta\sin\beta)}{g\cos\beta}$$

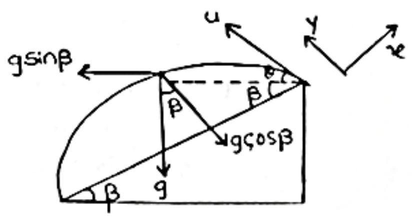
$$\therefore y=\frac{2u^2\sin(\theta-\beta)\cos\theta\sin\beta}{g\cos^2\beta}$$

**Motion in down the plane:**

$u_x=u\cos(\theta+\beta)$

$u_y=u\sin(\theta+\beta)$

$a_x=-g\sin\beta;\ a_y=-g\cos\beta$




$$y=u\sin(\theta+\beta)t-\frac{1}{2}(g\cos\beta)t^2$$

If $y = 0$ then $t = T$

$$u\sin(\theta+\beta)T = \frac{1}{2}(g\cos\beta)T^2$$

$$\boxed{\therefore T = \frac{2u\sin(\theta+\beta)}{g\cos\beta}}$$

Range $x = (u\cos\theta)\dfrac{2u\sin(\theta+\beta)}{g\cos\beta}$

(along horizontal)

$\rightarrow$ Range $R = \dfrac{x}{\cos\beta}$

$$\Rightarrow R = \frac{2u^2\sin(\theta+\beta)\cos\theta}{g\cos^2\beta}$$

$$R = \frac{u^2}{g}\left[\frac{2\sin(\theta+\beta)\cos\theta}{\cos^2\beta}\right] = \frac{u^2}{g}\left[\frac{\sin(\theta+\theta+\beta)+\sin\beta}{\cos^2\beta}\right]$$

$$R = \frac{u^2}{g}\left[\frac{\sin(2\theta+\beta)+\sin\beta}{1-\sin^2\beta}\right]$$

$$R = \frac{u^2}{g}\left[\frac{\sin(2\theta+\beta)+\sin\beta}{\cos^2\beta}\right]$$

If $R_{\max}$ then $\sin(2\theta+\beta) = 1$

$$R = \frac{u^2}{g}\left(\frac{1+\sin\beta}{1-\sin^2\beta}\right) \Rightarrow R_{\max} = \frac{u^2}{g(1-\sin\beta)}$$

# VVT Centres in Chennai

## ANNANAGAR

No. 162 1, 9th Main Rd, Shanthi Colony,
AI Block, Anna Nagar, Chennai, Tamil Nadu 600040

## ADYAR

Nibav Buildings, 4th & 5th Floor, No.23, Old No.11, Lattice Brg Rd,
Adyar, Chennai, Tamil Nadu 600020

## PALLIKARANAI

Plot 395, 1st Main Rd, Kamakoti Nagar, Pallikaranai,
Chennai, Tamil Nadu 600100

## MOGAPPAIR

M.G.R Adarsh Senior Secondary School,
11TH Block Mogappair East,Chennai - 600 037

info@vvtcoaching.com

www.vvtcoaching.com

+91 7200955557

+91 81221 22333

## Just Tap on Play Button